Half Life of Ba-137m

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Week: 7
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Partners: Alvin Modin, Lauren Hirai
Class: Intermediate Experimental Physics II
**Objective**

The purpose of this experiment was to observe the time-decay of the emission of radiation and to experimentally calculate the decay constant for $^{56}\text{Ba}^{137m}$.

**Theory**

Radioactive decay is not a deterministic function in time, which means that we cannot define precisely when a radioactive decomposition will take place. However, statistically speaking, we can define a probability per unit time for a radioactive decay. Using this time-dependent rate of decay, we can define a Time period within which the initial sample will have decayed to half of its initial amount. Given a sample with $N_0$ radioactive nuclei at time $t_0$, and shifting our time coordinates to $t_0 = 0$. The change in the number of radioactive nuclei (and therefore the radioactive nuclei that are stabilized in an infinitesimal time interval $dt$) is given by $dN = -\lambda N dt$. On integrating, this gives us:

$$
\ln N = -\lambda t + \ln N_0 \quad \Rightarrow \quad N = N_0 e^{-\lambda t}
$$

Since the decay is exponential with time, we can find the time in which the radioactive nuclei will reduce to half its initial number using:

$$
T_{1/2} = \frac{\ln(N_0/N)}{\lambda} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}
$$

**Experiment and Setup**

The radioactive source for this experiment contains a small quantity of $^{55}\text{Cs}^{137}$ which is radioactive and has a half-life of 30.1 years. This isotope of Cesium emits a $\beta^-$ particle and 94.6% of the time decays to a metastable state of Barium: $^{56}\text{Ba}^{137m}$. This Barium isotope emits a 0.662 MeV gamma ray photon when it decays to ground state. The half life of this Barium isotope is 153 s and reaches its equilibrium amount in 4-5 half-lives (10-12 min). When Barium is needed for the experiment, it can be flushed from the Cesium source using the “eluting” solution - this solution removes the radioactive Barium from the Cesium source without removing Cesium. Once Barium achieves its equilibrium state, more must be flushed out from the source to obtain results for this experiment. This makes it important to count decays almost immediately after drawing the barium solution.

**Procedure**

1. The Eluting solution was passed through the Cesium source in a controlled amount (seven drops), using a syringe, to produce a desirable quantity of metastable Barium.

2. The Barium source was placed under the counter window.

3. The Geiger counter was set up to measure radiation counts every 6 seconds (0.1 minute).
4. The counter reading was recorded every thirty seconds (from the start of one to the start of the other) for the 6 minutes that followed.

5. The strength of the Barium solution was measured for 1 min, and this strength was recorded.

6. The Barium solution was removed from the counter and a baseline “background radiation” was measured. Since the strength of the solution had already fallen significantly (more than 2 half-lives after preparation), the distance of the solution from the counter should make the radiation effect of the solution on the Geiger counter measurements negligible.

Data

The Voltage of the Geiger counter was 400V
After 1 min, the Geiger counter measurement for ambient radiation was 20
After 1 min, the Geiger counter measure for the Barium solution alone was 274

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<td>283</td>
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<td>249</td>
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Data Analysis

Figure 1: Natural log of Barium Geiger counter measurements against time, with errorbars to account for uncertainty and background radiation measurements
As expected, a certain amount of background radiation was recorded along with the measurements. The uncertainty in each measurement was estimated based on the initially recorded background radiation.

Since we know:

\[ \lambda = \frac{\ln(N_0/N_{1/2})}{T_{1/2}} = \frac{\ln(N_0/N_{\text{other}})}{t} \]

We can measure the decay constant for the Barium source using the time taken between recorded data points. This gave us a decay constant of 0.0045 s\(^{-1}\). From this we can find an experimental estimate for the half-life of Barium:

\[ T_{1/2} = \frac{0.693}{\lambda} = 154\, s \]

**Error Analysis**

All uncertainties were propagated using:

\[ \delta_y = \sqrt{\sum_{i=1}^{n} \left( \frac{\partial y}{\partial x_i} \right)^2 \delta_x^2_i} \]

So:

\[ \delta_{\ln(N)} = \sqrt{\left( \frac{1}{N} \right)^2 \delta_N^2} = \frac{1}{N} \delta_N \]

And, assuming the uncertainty in the time measurement was insignificant relative to the uncertainty in the measurement of the radiation counts, we can say:

\[ \delta_{\lambda} \approx \sqrt{\left( \frac{1}{N_1} \delta_N \right)^2 + \left( \frac{1}{N_2} \delta_N \right)^2} = \delta_N \sqrt{\left( \frac{1}{N_1} \right)^2 + \left( \frac{1}{N_2} \right)^2} = 0.0002 \leq \delta_N \]

From which the uncertainty in the half-life is:

\[ \delta_{T_{1/2}} = \frac{0.693}{\lambda^2} \delta_{\lambda} = 9\, s \]

**Questions**

*How far away does the source have to be before its presence makes no difference?*

Once the Barium solution was \( \approx 0.6\, m \) away from the Geiger counter, it’s presence made a negligible difference to the Geiger counter reading.

*How does the strength of the source fall off with distance? Discuss.*

The strength of the source (measured in terms of number of counts recorded by the geiger counter) falls with increase in distance from the source because the mean free path of Gamma rays of energy between 100 MeV and 1 GeV is \( \sim 10\, cm \)
Conclusions

In this experiment, we observed the decay of radiation with time and determined that the half-life of the sample was $\approx 154.6$ s which placed the accepted value of the half-life well within our calculated range of uncertainty. We confirmed that the data we collected corresponded to the expected exponential time-decay model and were able to study the uncertainty in the logarithmic plot against the line of best fit.

The background radiation in the lab was found to be 20 counts over a minute, remaining consistent over successive trials allowing us to use this as a basis for uncertainty estimation - if the background radiation was consistent, this number of counts could be used as a reasonable measure of uncertainty since this then accounts for the changes in background radiation, uncertainty in time-measurement, and apparatus uncertainty.